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Comments on regularization of identity based solutions in string field theory

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Abstract

We analyze the consistency of the recently proposed regularization of an identity based solution in open bosonic string field theory. We show that the equation of motion is satisfied when it is contracted with the regularized solution itself. Additionally, we propose a similar regularization of an identity based solution in the modified cubic superstring field theory.

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Contents

1	Introduction	2
2	Regularization of identity based solution in open bosonic string field theory	5
2.1	The kinetic term	6
2.2	The cubic term	7
2.3	\mathcal{L}_0 level expansion and Padé approximants	9
3	Regularization of identity based solution in the modified cubic superstring field theory	13
3.1	The kinetic term	14
3.2	The cubic term	16
3.3	\mathcal{L}_0 level expansion	17
4	Summary and discussion	18
A	Correlation functions	19
B	Padé approximant computations	21

1 Introduction

In a previous work [1], we have shown a prescription for computing identity based solutions in cubic-like string field theories [2, 3, 4]. Although these identity based solutions provide ambiguous result for the value of the vacuum energy [5], we noticed that the tractable Erler-Schnabl's solution of open bosonic string field theory [6] is related by a gauge transformation to a solution which is based on the identity string field. Moreover, we proved that the same is true in the case of the modified cubic superstring field theory, namely the regular solution of Gorbachev [7] is related by a gauge transformation to an identity based solution.

After performing the gauge transformation, the resulting Erler-Schnabl-type solutions were used to unambiguously compute the value of the vacuum energy. Nevertheless, it would be interesting to evaluate directly the vacuum energy using the identity based solutions, this kind of computation should be possible provide that we can find a consistent regularization scheme. Recently a proposal for regularizing an identity based solution in

open bosonic string field theory was developed in [8].

The regularized solution Ψ_λ was obtained by considering one-parameter families of classical solutions

$$\Psi_\lambda = U_\lambda Q U_\lambda^{-1} + U_\lambda \Psi_I U_\lambda^{-1}, \quad (1.1)$$

where $\Psi_I = c(1 - K)$ is the identity based solution found in [1] and

$$U_\lambda = 1 + \lambda c B K, \quad U_\lambda^{-1} = 1 - \lambda c B K \frac{1}{1 + \lambda K} \quad (1.2)$$

is an element of the gauge transformation [1, 8]. It has been shown that the resulting regularized solution Ψ_λ

$$\Psi_\lambda = (c + \lambda c K B c) \frac{1 + (\lambda - 1)K}{1 + \lambda K}, \quad (1.3)$$

correctly reproduces the value of the kinetic term

$$\langle \Psi_\lambda, Q \Psi_\lambda \rangle = -\frac{3}{\pi^2}, \quad (1.4)$$

and therefore 'assuming the equation of motion', the right value of the vacuum energy was reproduced. We have put assuming the equation of motion in quotation marks, since it remains as an important question if the regularization is consistent with the assumption that the equation of motion is satisfied when it is contracted with the solution itself, namely

$$\langle \Psi_\lambda, Q \Psi_\lambda \rangle + \langle \Psi_\lambda, \Psi_\lambda * \Psi_\lambda \rangle = 0. \quad (1.5)$$

From previous experiences in the past [9, 10, 11, 12, 13, 14], it is clear that there is a subtlety about this assumption because in general the solution is usually outside the Fock space [12]. For instance the twisted butterfly state [15, 16, 17] in vacuum string field theory [18] solves the equation of motion when contracted with any state in the Fock space, but it does not satisfy the equation of motion when contracted with the solution itself [11]. Therefore, for the case of the regularized solution Ψ_λ , it is important to test the validity of the assumption (1.5) and for this goal it is necessary to evaluate the cubic term of the string field theory action for the regularized solution Ψ_λ to check if the right value is reproduced ¹

$$\langle \Psi_\lambda, \Psi_\lambda * \Psi_\lambda \rangle = \frac{3}{\pi^2}. \quad (1.6)$$

¹Let us point out that a similar test of consistency was performed by Okawa [12], Fuchs, Kroyter [13] and Arroyo [14] for the original Schnabl's solution [19].

In this paper we compute the analytical value of the kinetic (1.4) and the cubic term (1.6) of the string field theory action for the regularized solution Ψ_λ and we show that the assumption of the equation of motion (1.5) was nevertheless correct². In addition to our analytic results, using Padé approximants [14] we numerically test equations (1.4) and (1.6) for the particular values $\lambda \rightarrow 0$ and $\lambda \rightarrow 1$ which correspond to the identity based and Erler-Schnabl's solution respectively. We would like to comment that there are cases where, naively, a one-parameter family of gauge transformations connects two solutions that cannot be gauge-equivalent (see for instance references [12, 13, 20, 21]). What can go wrong is that at some particular values of the parameter the gauge transformation becomes singular [20]. So at this point it is interesting to ask: whether this problem affects, or not, the gauge transformations (1.1) and (1.2). Do these gauge transformations become singular at some particular values of the parameter λ ? We will show that the gauge transformations (1.1) and (1.2) are well-defined for all values of the parameter λ belonging to the interval $[0, +\infty)$. These results provide a non trivial evidence for the consistency of the regularization proposed in [8].

Finally, we propose a similar regularization for an identity based solution in the modified cubic superstring field theory and, as in the bosonic case, we show that the regularized solution consistently reproduces the right value for the kinetic and cubic term and consequently for the vacuum energy. Our results show explicitly that how seemingly trivial identity based solutions, in open bosonic string field theory as well as in the modified cubic superstring field theory, can be consistently regularized to obtain well behaved solutions which precisely represent to the tachyon vacuum. Certainly it would be very interesting to extend these results to the case of the non-polynomial Berkovits WZW-type superstring field theory [22].

This paper is organized as follows. In section 2, we review the proposal for regularizing an identity based solution in open bosonic string field theory. We evaluate the kinetic and cubic term of the string field theory action for the regularized solution. It turns out that the value of the cubic term is correctly reproduced and therefore we prove the statement that the equation of motion is satisfied when it is contracted with the regularized solution itself. In section 3, we regularize an identity based solution in the modified cubic superstring field theory. As in the bosonic case, in order to prove the validity of the assumption that the equation of motion is satisfied when it is contracted with the regularized solution itself, we evaluate the kinetic and cubic term. In section 4, a summary and further directions of exploration are given. The appendix A is provided for explaining some details related to the computation of correlation functions. The appendix B is devoted to some explicit Padé approximants computations.

²Let us point out that the analytical value of the kinetic term (1.4) was already calculated in reference [8]. Nevertheless, for completeness reasons, in this paper we are going to review this computation.

2 Regularization of identity based solution in open bosonic string field theory

As derived in [1] using the methods of [23, 24], an identity based solution in open bosonic string field theory is given by

$$\Psi_I = c(1 - K) \quad (2.1)$$

where the basic string fields c and K (together with B) can be written, using the operator representation [19], as follows

$$K \rightarrow \frac{1}{2} \hat{\mathcal{L}} U_1^\dagger U_1 |0\rangle, \quad (2.2)$$

$$B \rightarrow \frac{1}{2} \hat{\mathcal{B}} U_1^\dagger U_1 |0\rangle, \quad (2.3)$$

$$c \rightarrow U_1^\dagger U_1 \tilde{c}(0) |0\rangle. \quad (2.4)$$

The operators $\hat{\mathcal{L}}$, $\hat{\mathcal{B}}$ and $\tilde{c}(0)$ are defined in the sliver frame [6]³, and they are related to the worldsheet energy-momentum tensor, the b and c ghosts fields respectively, for instance

$$\hat{\mathcal{L}} \equiv \mathcal{L}_0 + \mathcal{L}_0^\dagger = \oint \frac{dz}{2\pi i} (1 + z^2) (\arctan z + \operatorname{arccot} z) T(z), \quad (2.5)$$

$$\hat{\mathcal{B}} \equiv \mathcal{B}_0 + \mathcal{B}_0^\dagger = \oint \frac{dz}{2\pi i} (1 + z^2) (\arctan z + \operatorname{arccot} z) b(z), \quad (2.6)$$

while the operator $U_1^\dagger U_1$ in general is given by $U_r^\dagger U_r = e^{\frac{2-r}{2} \hat{\mathcal{L}}}$, so we have chosen $r = 1$, note that the string field $U_1^\dagger U_1 |0\rangle$ represents to the identity string field $1 \rightarrow U_1^\dagger U_1 |0\rangle$ [12, 19, 23, 24].

Using the operator representation (2.2)-(2.4) of the string fields K , B and c , we can show that these fields satisfy the algebraic relations

$$\{B, c\} = 1, \quad [B, K] = 0, \quad B^2 = c^2 = 0, \quad (2.7)$$

and have the following BRST variations

$$QK = 0, \quad QB = K, \quad Qc = cKc. \quad (2.8)$$

As it is shown in [1] the direct evaluation of the vacuum energy using the identity based solution (2.1) brings ambiguous result. This phenomenon, as it was noted in [8], is

³Remember that a point in the upper half plane z is mapped to a point in the sliver frame \tilde{z} via the conformal mapping $\tilde{z} = \frac{2}{\pi} \arctan z$. Note that we are using the convention of [6] for the conformal mapping.

due to the fact that a naive evaluation of the classical action in terms of CFT methods tends to be indefinite since it corresponds to a correlator on vanishing strip. Recently this problem was overcome and a proposal for regularizing the identity based solution (2.1) has been developed in [8].

The regularized solution Ψ_λ is obtained by considering one-parameter families of classical solutions

$$\Psi_\lambda = U_\lambda Q U_\lambda^{-1} + U_\lambda \Psi_I U_\lambda^{-1}, \quad (2.9)$$

where Ψ_I is the identity based solution (2.1) and

$$U_\lambda = 1 + \lambda c B K, \quad U_\lambda^{-1} = 1 - \lambda c B K \frac{1}{1 + \lambda K} \quad (2.10)$$

is an element of the gauge transformation [1, 8]. Using (2.1), (2.9) and (2.10), it is almost easy to derive the following regularized solution

$$\Psi_\lambda = c(1 + \lambda K) B c \frac{1 + (\lambda - 1)K}{1 + \lambda K}. \quad (2.11)$$

Note that this regularized solution interpolates between the identity based solution (2.1) which corresponds to the case $\lambda \rightarrow 0$, and the Erler-Schnabl's solution [6] which corresponds to the case $\lambda \rightarrow 1$. In the next subsection we are going to evaluate the kinetic term for the regularized solution, and it will be shown that its value does not depend on the parameter λ .

2.1 The kinetic term

In this subsection, we are going to evaluate the kinetic term of the string field theory action for the regularized solution Ψ_λ ⁴

$$\langle \Psi_\lambda, Q \Psi_\lambda \rangle. \quad (2.12)$$

Since the regularized solution (2.11) can be written as an expression containing an exact BRST term

$$\Psi_\lambda = c \frac{1 + (\lambda - 1)K}{1 + \lambda K} + Q \left\{ \lambda B c \frac{1 + (\lambda - 1)K}{1 + \lambda K} \right\}, \quad (2.13)$$

⁴Let us point out that, in the bosonic case, the analytical calculation of the kinetic term was already performed in [8]. Nevertheless, for completeness reasons, in this subsection we are going to review this computation.

the computation of the kinetic term (2.12) can be simplified to the evaluation of the following correlator

$$\begin{aligned}
\langle \Psi_\lambda, Q\Psi_\lambda \rangle &= \left\langle c \frac{1 + (\lambda - 1)K}{1 + \lambda K} cK c \frac{1 + (\lambda - 1)K}{1 + \lambda K} \right\rangle \\
&= \int_0^\infty \int_0^\infty dt_1 dt_2 e^{-t_1 - t_2} \langle c(1 + (\lambda - 1)K) \Omega^{\lambda t_1} cK c(1 + (\lambda - 1)K) \Omega^{\lambda t_2} \rangle \\
&= \int_0^\infty \int_0^\infty dt_1 dt_2 e^{-t_1 - t_2} \left(1 + \frac{1 - \lambda}{\lambda} \partial_{t_1} \right) \left(1 + \frac{1 - \lambda}{\lambda} \partial_{t_2} \right) \langle c \Omega^{\lambda t_1} cK c \Omega^{\lambda t_2} \rangle.
\end{aligned} \tag{2.14}$$

Using the expression for the correlator $\langle c \Omega^{\lambda t_1} cK c \Omega^{\lambda t_2} \rangle$ (given in the appendix), applying the change of variables as in [6] $t_1 \rightarrow uv$, $t_2 \rightarrow u(1 - v)$ and performing the v integral, we get from (2.14)

$$\begin{aligned}
\langle \Psi_\lambda, Q\Psi_\lambda \rangle &= -\frac{1}{2\pi^2} \int_0^\infty du e^{-u} [6(\lambda - 1)^2 u - 6(\lambda - 1)\lambda u^2 + \lambda^2 u^3] \\
&= -\frac{3}{\pi^2}.
\end{aligned} \tag{2.15}$$

Therefore, as it was previously commented, the value of the kinetic term does not depend on the parameter λ . At this stage, we can safely take the limit $\lambda \rightarrow 0$ which corresponds to the identity based solution.

If we assume the validity of the equation of motion when contracted with the regularized solution itself, it is clear that the value of the vacuum energy can be correctly reproduced [8]. Nevertheless, there is a subtlety about this assumption because in general the solution is usually outside the Fock space [12]. Therefore it is crucially important to know whether or not the equation of motion is satisfied when it is contracted with the regularized solution itself. To prove the correctness of this statement, it is necessary to evaluate the cubic term of the string field theory action.

2.2 The cubic term

In this subsection, we are going to evaluate the cubic term of the string field theory action for the regularized solution

$$\langle \Psi_\lambda, \Psi_\lambda * \Psi_\lambda \rangle. \tag{2.16}$$

Since the regularized solution (2.11) can be written as an expression containing two

terms

$$\Psi_\lambda = \Psi_1 + \Psi_2, \quad (2.17)$$

$$\Psi_1 = c \frac{1 + (\lambda - 1)K}{1 + \lambda K}, \quad (2.18)$$

$$\Psi_2 = \lambda c B K c \frac{1 + (\lambda - 1)K}{1 + \lambda K}, \quad (2.19)$$

the calculation of the cubic term (2.16) can be reduced to the evaluation of the following correlators

$$\langle \Psi_\lambda, \Psi_\lambda * \Psi_\lambda \rangle = \langle \Psi_1, \Psi_1 * \Psi_1 \rangle + 3\langle \Psi_2, \Psi_2 * \Psi_1 \rangle + 3\langle \Psi_2, \Psi_1 * \Psi_1 \rangle + \langle \Psi_2, \Psi_2 * \Psi_2 \rangle, \quad (2.20)$$

each term on the right hand side (RHS) of (2.20) is given by

$$\langle \Psi_1, \Psi_1 * \Psi_1 \rangle = \int_0^\infty \int_0^\infty \int_0^\infty dt_1 dt_2 dt_3 e^{-t_1 - t_2 - t_3} \mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_3 \langle c\Omega^{\lambda t_1} c\Omega^{\lambda t_2} c\Omega^{\lambda t_3} \rangle, \quad (2.21)$$

$$\langle \Psi_2, \Psi_2 * \Psi_1 \rangle = \lambda^2 \int_0^\infty \int_0^\infty \int_0^\infty dt_1 dt_2 dt_3 e^{-t_1 - t_2 - t_3} \mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_3 \langle c B K c \Omega^{\lambda t_1} c B K c \Omega^{\lambda t_2} c \Omega^{\lambda t_3} \rangle, \quad (2.22)$$

$$\langle \Psi_2, \Psi_1 * \Psi_1 \rangle = \lambda \int_0^\infty \int_0^\infty \int_0^\infty dt_1 dt_2 dt_3 e^{-t_1 - t_2 - t_3} \mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_3 \langle c B K c \Omega^{\lambda t_1} c \Omega^{\lambda t_2} c \Omega^{\lambda t_3} \rangle, \quad (2.23)$$

where the differential operators \mathcal{D}_1 , \mathcal{D}_2 and \mathcal{D}_3 are defined as

$$\mathcal{D}_i \equiv 1 + \frac{1 - \lambda}{\lambda} \partial_{t_i}, \quad i = 1, 2, 3. \quad (2.24)$$

Since the string field Ψ_2 defined in (2.19) can be written as an exact BRST term (2.13), the last term on the RHS of (2.20) gives vanishing result.

Using the expression for the correlators $\langle c\Omega^{\lambda t_1} c\Omega^{\lambda t_2} c\Omega^{\lambda t_3} \rangle$, $\langle c B K c \Omega^{\lambda t_1} c B K c \Omega^{\lambda t_2} c \Omega^{\lambda t_3} \rangle$ and $\langle c B K c \Omega^{\lambda t_1} c \Omega^{\lambda t_2} c \Omega^{\lambda t_3} \rangle$ (given in the appendix) into equations (2.21)–(2.23), applying the change of variables as in [25] $t_1 \rightarrow uv_1$, $t_2 \rightarrow uv_2$, $t_3 \rightarrow u(1 - v_1 - v_2)$ and performing the v_1, v_2 integral, we obtain from (2.20)

$$\begin{aligned} \langle \Psi_\lambda, \Psi_\lambda * \Psi_\lambda \rangle &= \frac{1}{8\pi^4} \int_0^\infty du e^{-u} \left[24(-15 + \pi^2)(\lambda - 1)^3 - 24(-15 + \pi^2)(\lambda - 1)^2(5\lambda - 2)u \right. \\ &\quad + 36(\lambda - 1)(3\pi^2\lambda^2 - 50\lambda^2 - 2\pi^2\lambda + 40\lambda - 5)u^2 + \lambda^2(2\pi^2\lambda - 75\lambda + 45)u^4 \\ &\quad \left. - 4\lambda(7\pi^2\lambda^2 - 150\lambda^2 - 6\pi^2\lambda + 180\lambda - 45)u^3 + 3\lambda^3 u^5 \right] \\ &= \frac{3}{\pi^2}. \end{aligned} \quad (2.25)$$

We see that the value of the cubic term does not depend on the parameter λ . This result (2.25) proves the statement that the equation of motion is satisfied when it is contracted with the regularized solution itself.

2.3 \mathcal{L}_0 level expansion and Padé approximants

Although we have an analytic result for the value of the kinetic (2.15) and cubic term (2.25), we would like to confirm our calculation by using the \mathcal{L}_0 level expansion of the solution. A numerical method based on the \mathcal{L}_0 level expansion of the solution was developed in references [6, 14, 19], where the vacuum energy for the original Schnabl's solution [19] was represented as a formal sum of an asymptotic series which was resumed using Padé approximants. In this subsection, using this \mathcal{L}_0 level expansion scheme, we are going to numerically test equations (2.15) and (2.25) for the particular values $\lambda \rightarrow 0$ and $\lambda \rightarrow 1$ which correspond to the identity based (2.1) and Erler-Schnabl's solution [6] respectively.

In order to evaluate the kinetic and cubic term of the string field theory action in the \mathcal{L}_0 level expansion scheme, let us write the regularized solution (2.11) in terms of \mathcal{L}_0 eigenstates

$$\Psi_\lambda = \sum_{n,p} f_{n,p}(\lambda) (\mathcal{L}_0 + \mathcal{L}_0^\dagger)^n \tilde{c}_p |0\rangle + \sum_{n,p,q} f_{n,p,q}(\lambda) (\mathcal{B}_0 + \mathcal{B}_0^\dagger) (\mathcal{L}_0 + \mathcal{L}_0^\dagger)^n \tilde{c}_p \tilde{c}_q |0\rangle, \quad (2.26)$$

$$\begin{aligned} f_{n,p}(\lambda) = & 2^{-n} \frac{(\lambda - 1)}{\lambda} \left(\frac{1}{n!} + \frac{\lambda}{(n-1)!} \right) \delta_{1,p} \\ & + \frac{2^{-n+p-1}}{\lambda} \sum_{k=0}^n \frac{(-1)^{n-k} \lambda^{n-k-p+1} (n-k-p+1)!}{k!(n-k)!} \\ & + 2^{-n+p-1} \sum_{k=0}^{n-1} \frac{(-1)^{n-k-1} \lambda^{n-k-p} (n-k-p)!}{k!(n-k-1)!}, \end{aligned} \quad (2.27)$$

$$\begin{aligned} f_{n,p,q}(\lambda) = & \frac{2^{-n-2}(1-\lambda)}{n!} (\delta_{0,q} \delta_{1,p} - \delta_{0,p} \delta_{1,q}) \\ & + 2^{-n+p+q-3} (q-p) \sum_{k=0}^n \frac{(-1)^{n-k} \lambda^{n-k-p-q+1} (n-k-p-q+1)!}{k!(n-k)!}. \end{aligned} \quad (2.28)$$

As it is described in [6, 14, 19], we start by replacing the solution Ψ_λ with $z^{\mathcal{L}_0} \Psi_\lambda$ in the \mathcal{L}_0 level truncation scheme, so that states in the \mathcal{L}_0 level expansion of the solution will acquire different integer powers of z at different levels. As we are going to see, the parameter z is needed because we need to express the kinetic and cubic term as a formal power series expansion if we want to use Padé approximants. After doing our calculations, we will simply set $z = 1$. Plugging the \mathcal{L}_0 level expansion of the regularized solution (2.26)

into the kinetic term we obtain

$$\begin{aligned}
\langle \Psi_\lambda, z^{\mathcal{L}_0^\dagger} Q z^{\mathcal{L}_0} \Psi_\lambda \rangle = & -\frac{4}{\pi^2 z^2} + \frac{(\pi^2 - 4)(2\lambda - 1)}{2\pi^2} + \frac{\pi^2(\lambda^2 - 2\lambda^3)}{8} z^2 + \frac{\pi^2(5\lambda^4 - 4\lambda^3 + \lambda^2)}{4} z^3 \\
& + \frac{\pi^2(2\pi^2\lambda^5 - 180\lambda^5 - \pi^2\lambda^4 + 174\lambda^4 - 72\lambda^3 + 12\lambda^2)}{32} z^4 \\
& + \frac{\pi^2(-7\pi^2\lambda^6 + 210\lambda^6 + 5\pi^2\lambda^5 - 222\lambda^5 - \pi^2\lambda^4 + 114\lambda^4 - 32\lambda^3 + 4\lambda^2)}{8} z^5 \\
& + \dots .
\end{aligned} \tag{2.29}$$

Given this formal power series expansion (2.29) of the kinetic term, we are going to resum the series using Padé approximants for some particular values of the parameter λ . Basically the numerical method based on Padé approximants tell us to match the power series expansion coefficients of a given rational function $P_{2+N}^M(z)$ with those of the kinetic term (2.29). The details of these computations can be found in appendix B.

The result of our calculations is summarized in table 2.1. In the first column we show the normalized value of the kinetic term $\frac{\pi^2}{3} \langle \Psi_\lambda, z^{\mathcal{L}_0^\dagger} Q z^{\mathcal{L}_0} \Psi_\lambda \rangle$ for the particular value of the parameter $\lambda \rightarrow 0$ which corresponds to the identity based solution. In the second column we show the normalized value of the kinetic term for the particular value of the parameter $\lambda \rightarrow 1$ which corresponds to the Erler-Schnabl's solution⁵. As we can see from table 2.1 the value of the kinetic term computed numerically using Padé approximants nicely confirm the analytic result (2.15).

Table 2.1: The Padé approximation for the normalized value of the kinetic term $\frac{\pi^2}{3} \langle \Psi_\lambda, z^{\mathcal{L}_0^\dagger} Q z^{\mathcal{L}_0} \Psi_\lambda \rangle$ evaluated at $z = 1$. The first column corresponds to P_{2+n}^n Padé approximation for the value of the parameter $\lambda \rightarrow 0$, while in the second column we show the case $\lambda \rightarrow 1$. The label n corresponds to the power of z in the series (2.29). At each stage of our computations we truncate the series up to the order z^{2n-2} .

	$P_{2+n}^n(\lambda \rightarrow 0)$ Padé approximation	$P_{2+n}^n(\lambda \rightarrow 1)$ Padé approximation
$n = 0$	-1.333333333333	-1.333333333333
$n = 2$	-2.311600733514	-1.143337106188
$n = 4$	-1.494531194598	-0.898882661597
$n = 6$	-1.077721474044	-1.042410506615
$n = 8$	-1.606155676411	-0.996478424643
$n = 10$	-0.951428845113	-0.995773031227
$n = 12$	-1.008770444432	-0.999000106158
$n = 14$	-1.001300960887	-0.999417017099
$n = 16$	-1.005286583496	-0.999198792613

⁵Actually the numerical computation of the kinetic term using Padé approximants for the particular value of the parameter $\lambda \rightarrow 1$ was already performed in reference [6].

To evaluate the cubic term using its formal power series expansion in z by means of Padé approximants, we follow the same steps developed in the case of the kinetic term. First we plug the \mathcal{L}_0 level expansion of the regularized solution (2.26) into the expression for the cubic term

$$\begin{aligned}
\langle \Psi_\lambda, z^{\mathcal{L}_0^\dagger} (z^{\mathcal{L}_0} \Psi_\lambda) * (z^{\mathcal{L}_0} \Psi_\lambda) \rangle &= \frac{81\sqrt{3}}{8\pi^3} \frac{1}{z^3} + \frac{27(-3\sqrt{3} + \pi)\lambda}{4\pi^3 z^2} \\
&+ \frac{4\sqrt{3}\pi^2(2\lambda^2 - 6\lambda + 3) + 27\sqrt{3}(2\lambda^2 + 6\lambda - 3) - 36\pi\lambda^2}{8\pi^3 z} \\
&+ \frac{\pi^3(8 - 12\lambda) - 36\sqrt{3}\pi^2(\lambda - 1)^2 + 81\pi\lambda(2\lambda - 1) + 81\sqrt{3}(1 - 3\lambda^2)}{36\pi^3} \\
&+ \frac{2\pi\lambda(3\lambda - 2)}{27\sqrt{3}} z + \frac{4\pi\lambda(3(-6\sqrt{3} + \pi)\lambda^2 + (24\sqrt{3} - 2\pi)\lambda - 9\sqrt{3})}{729} z^2 \\
&+ \dots
\end{aligned} \tag{2.30}$$

Given the formal power series expansion (2.30), we are able to evaluate the cubic term using Padé approximants. We match the power series expansion coefficients of a given rational function $P_{3+N}^M(z)$ with those of the cubic term (2.30). The result of our computations is summarized in table 2.2. In the first column we show the normalized value of the cubic term $\frac{\pi^2}{3} \langle \Psi_\lambda, z^{\mathcal{L}_0^\dagger} (z^{\mathcal{L}_0} \Psi_\lambda) * (z^{\mathcal{L}_0} \Psi_\lambda) \rangle$ for the particular value of the parameter $\lambda \rightarrow 0$ which corresponds to the identity based solution. In the second column we show the normalized value of the cubic term for the particular value of the parameter $\lambda \rightarrow 1$ which corresponds to the Erler-Schnabl's solution. As we can see from table 2.2 the value of the cubic term computed numerically using Padé approximants nicely confirm the analytic result (2.25).

As it was mentioned in the introduction section, at this point we would like to argue that the gauge transformations (1.1) and (1.2) are well-defined for all values of the parameter λ belonging to the interval $[0, +\infty)$. In order to develop the arguments to prove this statement, let us review the discussion given for the case of the original Schnabl's solution [19]. It is known that Schnabl's solution can be written as the limit, $\lambda \rightarrow 1$, of the following pure gauge form [12, 13, 20]

$$\Psi_\lambda^S = \Gamma_\lambda Q \Gamma_\lambda^{-1}, \tag{2.31}$$

where S stands for Schnabl's solution and

$$\Gamma_\lambda = 1 - \lambda\Phi, \quad \Gamma_\lambda^{-1} = \frac{1}{1 - \lambda\Phi} \tag{2.32}$$

with

$$\Phi = B_1^L \tilde{c}_1 |0\rangle. \tag{2.33}$$

Table 2.2: The Padé approximation for the normalized value of the cubic term $\frac{\pi^2}{3} \langle \Psi_\lambda, z^{\mathcal{L}_0^\dagger} (z^{\mathcal{L}_0} \Psi_\lambda) * (z^{\mathcal{L}_0} \Psi_\lambda) \rangle$ evaluated at $z = 1$. The first column corresponds to $P_{3+n/2}^{n/2}$ Padé approximation for the value of the parameter $\lambda \rightarrow 0$, while in the second column we show the case $\lambda \rightarrow 1$. The label n corresponds to the power of z in the series (2.30). At each stage of our computations we truncate the series up to the order z^{n-3} .

	$P_{3+n/2}^{n/2}(\lambda \rightarrow 0)$ Padé approximation	$P_{3+n/2}^{n/2}(\lambda \rightarrow 1)$ Padé approximation
$n = 0$	1.860735022048	1.860735022048
$n = 2$	1.860735022048	0.860998808763
$n = 4$	2.221490574460	0.839819361621
$n = 6$	2.051478161167	0.883381212050
$n = 8$	0.893773019606	0.967208479640
$n = 10$	1.088076700691	0.983453704434
$n = 12$	1.038107274783	0.964647638538
$n = 14$	1.005610433784	0.995649817299
$n = 16$	1.003669862242	0.997027139055

When $\lambda < 1$ the string fields Ψ_λ^S and Γ_λ^{-1} are well-defined in the level-expansion and the solution Ψ_λ^S is a pure-gauge solution with zero energy [12, 13, 20, 21]. Obviously, Schnabl's solution cannot be a pure-gauge solution since the energy of such a solution would have to be zero in contradiction with the proven Sen's first conjecture. It is therefore interesting to understand how the solution ceases to be a pure-gauge in the limit $\lambda \rightarrow 1$.

It turns out that the gauge transformation (2.32) becomes singular at $\lambda = 1$ [20]. To see how this can happen, let us expand the string field Γ_λ^{-1} in the \mathcal{L}_0 basis

$$\begin{aligned}
\Gamma_\lambda^{-1} = & -\frac{2-\lambda}{2(\lambda-1)}|0\rangle - \frac{\lambda}{2(\lambda-1)}\hat{\mathcal{B}}\tilde{c}_1|0\rangle + \frac{\lambda^2-4\lambda+2}{4(\lambda-1)^2}\hat{\mathcal{L}}|0\rangle - \frac{\lambda^2}{4(\lambda-1)^2}\hat{\mathcal{B}}\tilde{c}_0|0\rangle \\
& - \frac{\lambda^2}{4(\lambda-1)^2}\hat{\mathcal{L}}\hat{\mathcal{B}}\tilde{c}_1|0\rangle - \frac{\lambda^2(\lambda+1)}{8(\lambda-1)^3}\hat{\mathcal{B}}\tilde{c}_{-1}|0\rangle + \frac{\lambda^3-7\lambda^2+6\lambda-2}{16(\lambda-1)^3}\hat{\mathcal{L}}^2|0\rangle \\
& - \frac{\lambda^2(\lambda+1)}{8(\lambda-1)^3}\hat{\mathcal{L}}\hat{\mathcal{B}}\tilde{c}_0|0\rangle - \frac{\lambda^2(\lambda+1)}{16(\lambda-1)^3}\hat{\mathcal{L}}^2\hat{\mathcal{B}}\tilde{c}_1|0\rangle + \dots,
\end{aligned} \tag{2.34}$$

where the dots stand for terms of higher \mathcal{L}_0 -level. From the expansion (2.34) it is clear that the string field Γ_λ^{-1} is not well-defined at $\lambda = 1$. By this method, it is possible to show the presence of poles in the definition of the gauge transformations, nevertheless the expansion (2.34) does not tell us much about the interval where the parameter λ should belong. In the reference [21] it was argued that Schnabl's solution Ψ_λ^S has a well-defined Fock space expression if the parameter λ belongs to the interval $[-1, +1)$. The proof is based on the convergence properties of the coefficients which appear when we expand Ψ_λ^S in the usual Virasoro L_0 basis.

Coming back to the case of the gauge transformations (1.1) and (1.2), to determine

the interval where the parameter λ should belong, we will employ the same arguments developed for the case of the original Schnabl's solution Ψ_λ^S . So let us first check that if potential singularities can arise in the definition of the gauge transformations (1.1) and (1.2). The place where potential singularities can arise is in the definition of the inverse of the string field U_λ . To see if this problem can happen, let us expand the string field U_λ^{-1} in the \mathcal{L}_0 basis

$$\begin{aligned} U_\lambda^{-1} = & +|0\rangle + \left(\frac{1}{2} - \frac{\lambda}{4}\right)\hat{\mathcal{L}}|0\rangle - \frac{\lambda}{4}\hat{\mathcal{B}}\tilde{c}_0|0\rangle + \frac{\lambda}{4}\hat{\mathcal{L}}\hat{\mathcal{B}}\tilde{c}_1|0\rangle - \frac{\lambda^2}{4}\hat{\mathcal{B}}\tilde{c}_{-1}|0\rangle + \left(\frac{\lambda^2}{4} - \frac{\lambda}{8}\right)\hat{\mathcal{L}}\hat{\mathcal{B}}\tilde{c}_0|0\rangle \\ & + \left(\frac{\lambda^2}{8} - \frac{\lambda}{8} + \frac{1}{8}\right)\hat{\mathcal{L}}^2|0\rangle + \left(\frac{\lambda}{8} - \frac{\lambda^2}{8}\right)\hat{\mathcal{L}}^2\hat{\mathcal{B}}\tilde{c}_1|0\rangle + \dots \end{aligned} \quad (2.35)$$

From this expansion (2.35), we see that the string field U_λ^{-1} does not have poles neither at $\lambda = 1$ or $\lambda = 0$. In the same way, we can also see that the \mathcal{L}_0 -level expansion of the regularized solution (1.1)

$$\begin{aligned} \Psi_\lambda = & +\tilde{c}_1|0\rangle + \frac{1}{2}\tilde{c}_0|0\rangle - \frac{\lambda}{2}\hat{\mathcal{B}}\tilde{c}_1\tilde{c}_0|0\rangle + \frac{\lambda}{2}\hat{\mathcal{L}}\tilde{c}_1|0\rangle + \frac{\lambda}{2}\tilde{c}_{-1}|0\rangle - \frac{\lambda}{2}\hat{\mathcal{B}}\tilde{c}_1\tilde{c}_{-1}|0\rangle \\ & + \left(\frac{1}{4} - \frac{\lambda}{4}\right)\hat{\mathcal{L}}\tilde{c}_0|0\rangle + \left(\frac{\lambda}{4} - \frac{1}{8}\right)\hat{\mathcal{L}}^2\tilde{c}_1|0\rangle + \frac{3\lambda^2}{4}\tilde{c}_{-2}|0\rangle - \frac{3\lambda^2}{4}\hat{\mathcal{B}}\tilde{c}_1\tilde{c}_{-2}|0\rangle \\ & - \frac{\lambda^2}{4}\hat{\mathcal{B}}\tilde{c}_0\tilde{c}_{-1}|0\rangle + \left(\frac{\lambda}{4} - \frac{\lambda^2}{2}\right)\hat{\mathcal{L}}\tilde{c}_{-1}|0\rangle + \left(\frac{\lambda^2}{8} - \frac{\lambda}{8} + \frac{1}{16}\right)\hat{\mathcal{L}}^2\tilde{c}_0|0\rangle + \dots \end{aligned} \quad (2.36)$$

does not have poles neither at $\lambda = 1$ or $\lambda = 0$.

By using this method, we have just shown the absence of poles in the definition of the gauge transformations (1.1) and (1.2). It remains the question about the interval where the parameter λ should belong. As in the case of the original Schnabl's solution [21] to provide an answer to this question, we need to expand the regularized solution Ψ_λ in the Virasoro L_0 basis. It turns out that the coefficients of the L_0 -level expansion of the regularized solution (1.1) are given by sums of integrals of the form

$$\int_0^\infty e^{-t}(1+\lambda t)^m \cos^2\left(\frac{\pi}{2}\frac{\lambda t}{1+\lambda t}\right) \tan^n\left(\frac{\pi}{2}\frac{\lambda t}{1+\lambda t}\right), \quad (2.37)$$

for $m = 2, 0, -2, -4, \dots$ and $n \in \mathbb{N}_0$. These integrals are convergent provide that the parameter λ belongs to the interval $[0, +\infty)$. Therefore, as it was claimed, the gauge transformations (1.1) and (1.2) are well-defined for all positive values of the parameter λ .

3 Regularization of identity based solution in the modified cubic superstring field theory

In this section, we extend our previous results in order to regularize an identity based solution in the modified cubic superstring field theory. In the superstring case, in addition

to the basic string fields K , B and c , we need to include the super-reparametrization ghost field γ which, in the operator representation, is given by [26]

$$\gamma \rightarrow U_1^\dagger U_1 \tilde{\gamma}(0)|0\rangle. \quad (3.1)$$

Let us remember that in the superstring case the basic string fields K , B , c and γ satisfy the algebraic relations [7, 26]

$$\begin{aligned} \{B, c\} = 1, \quad [B, K] = 0, \quad B^2 = c^2 = 0, \\ \partial c = [K, c], \quad \partial \gamma = [K, \gamma], \quad [c, \gamma] = 0, \quad [B, \gamma] = 0, \end{aligned} \quad (3.2)$$

and have the following BRST variations

$$QK = 0, \quad QB = K, \quad Qc = cKc - \gamma^2, \quad Q\gamma = c\partial\gamma - \frac{1}{2}\gamma\partial c. \quad (3.3)$$

Employing these basic string fields, we can construct the following identity based solution

$$\Psi_I = (c + B\gamma^2)(1 - K) \quad (3.4)$$

which formally satisfies the equation of motion $Q\Psi_I + \Psi_I^2 = 0$, where in this case Q is the BRST operator of the open Neveu-Schwarz superstring theory.

As in the bosonic case, the direct evaluation of the vacuum energy using the identity based solution (3.4) brings ambiguous result. Therefore before computing some gauge invariants, such as the vacuum energy, first we need to regularize our identity based solution. Using the same procedure developed in the previous section, we show that a well behaved regularized solution Ψ_λ can be derived from our identity based solution (3.4) by performing a gauge transformation

$$\begin{aligned} \Psi_\lambda &= U_\lambda(Q + \Psi_I)U_\lambda^{-1} \\ &= \left[\lambda cBK + 1\right] \left(Q + (c + B\gamma^2)(1 - K)\right) \left[1 - \lambda cBK \frac{1}{1 + \lambda K}\right] \\ &= (c + \lambda cKBc + B\gamma^2) \frac{1 + (\lambda - 1)K}{1 + \lambda K}. \end{aligned} \quad (3.5)$$

Note that this regularized solution interpolates between the identity based solution (3.4) which corresponds to the case $\lambda \rightarrow 0$, and the Gorbachev's solution [7] which corresponds to the case $\lambda \rightarrow 1$. In the next subsection we are going to evaluate the kinetic term for the regularized solution, and it will be shown that its value does not depend on the parameter λ .

3.1 The kinetic term

In this subsection, we are going to evaluate the kinetic term of the modified cubic superstring field theory action for the regularized solution Ψ_λ

$$\langle\langle\Psi_\lambda, Q\Psi_\lambda\rangle\rangle. \quad (3.6)$$

The inner product $\langle\langle \cdot, \cdot \rangle\rangle$ is the standard BPZ inner product with the difference that we must insert the operator Y_{-2} at the open string midpoint. The operator Y_{-2} can be written as the product of two inverse picture changing operators $Y_{-2} = Y(i)Y(-i)$, where $Y(z) = -\partial\xi e^{-2\phi}c(z)$.

In order to simplify the computations, let us write the regularized solution (3.5) as an expression containing two terms

$$\Psi_\lambda = \Psi_1 + \Psi_2, \quad (3.7)$$

$$\Psi_1 = c \frac{1 + (\lambda - 1)K}{1 + \lambda K}, \quad (3.8)$$

$$\Psi_2 = (\lambda c B K c + B \gamma^2) \frac{1 + (\lambda - 1)K}{1 + \lambda K}. \quad (3.9)$$

Replacing equations (3.7)–(3.9) into the expression for the kinetic term (3.6), we obtain

$$\langle\langle \Psi_\lambda, Q \Psi_\lambda \rangle\rangle = \langle\langle \Psi_1, Q \Psi_1 \rangle\rangle + 2\langle\langle \Psi_1, Q \Psi_2 \rangle\rangle + \langle\langle \Psi_2, Q \Psi_2 \rangle\rangle, \quad (3.10)$$

each term on the RHS of (3.10) is given by

$$\langle\langle \Psi_1, Q \Psi_1 \rangle\rangle = - \int_0^\infty \int_0^\infty dt_1 dt_2 e^{-t_1 - t_2} \mathcal{D}_1 \mathcal{D}_2 \langle\langle c \Omega^{\lambda t_1} \gamma^2 \Omega^{\lambda t_2} \rangle\rangle, \quad (3.11)$$

$$\langle\langle \Psi_1, Q \Psi_2 \rangle\rangle = 2(1 - \lambda) \int_0^\infty \int_0^\infty dt_1 dt_2 e^{-t_1 - t_2} \mathcal{D}_1 \mathcal{D}_2 \langle\langle c \Omega^{\lambda t_1} c B K \gamma^2 \Omega^{\lambda t_2} \rangle\rangle, \quad (3.12)$$

$$\langle\langle \Psi_2, Q \Psi_2 \rangle\rangle = 2\lambda(1 - \lambda) \int_0^\infty \int_0^\infty dt_1 dt_2 e^{-t_1 - t_2} \mathcal{D}_1 \mathcal{D}_2 \langle\langle c B K c \Omega^{\lambda t_1} c B K \gamma^2 \Omega^{\lambda t_2} \rangle\rangle. \quad (3.13)$$

The correlators $\langle\langle c \Omega^{\lambda t_1} \gamma^2 \Omega^{\lambda t_2} \rangle\rangle$, $\langle\langle c \Omega^{\lambda t_1} c B K \gamma^2 \Omega^{\lambda t_2} \rangle\rangle$ and $\langle\langle c B K c \Omega^{\lambda t_1} c B K \gamma^2 \Omega^{\lambda t_2} \rangle\rangle$ can be computed using the methods given in the appendix. Plugging the expression for these correlators into equations (3.11)–(3.13), applying the change of variables as in [6] $t_1 \rightarrow uv$, $t_2 \rightarrow u(1 - v)$ and performing the v integral, we obtain from (3.10)

$$\begin{aligned} \langle\langle \Psi_\lambda, Q \Psi_\lambda \rangle\rangle &= -\frac{1}{2\pi^2} \int_0^\infty du e^{-u} [6(\lambda - 1)^2 u - 6(\lambda - 1)\lambda u^2 + \lambda^2 u^3] \\ &= -\frac{3}{\pi^2}. \end{aligned} \quad (3.14)$$

Therefore, as in the bosonic case, the value of the kinetic term does not depend on the parameter λ . At this stage, we can safely take the limit $\lambda \rightarrow 0$ which corresponds to the identity based solution.

If we assume the validity of the equation of motion when contracted with the solution itself, it is clear that the value of the vacuum energy can be correctly reproduced. Nevertheless it is important to test whether or not the equation of motion is satisfied when it is

contracted with the regularized solution itself. To prove the correctness of this statement, it is necessary to compute explicitly the cubic term of the modified cubic superstring field theory action.

3.2 The cubic term

In this subsection, we are going to evaluate the cubic term of the modified cubic superstring field theory action for the regularized solution

$$\langle\langle\Psi_\lambda, \Psi_\lambda * \Psi_\lambda\rangle\rangle. \quad (3.15)$$

Since the regularized solution (3.5) can be written as an expression containing two terms (3.8) and (3.9), the calculation of the cubic term (3.15) can be reduced to the evaluation of the following correlators

$$\langle\langle\Psi_\lambda, \Psi_\lambda * \Psi_\lambda\rangle\rangle = \langle\langle\Psi_1, \Psi_1 * \Psi_1\rangle\rangle + 3\langle\langle\Psi_2, \Psi_1 * \Psi_1\rangle\rangle + 3\langle\langle\Psi_2, \Psi_2 * \Psi_1\rangle\rangle + \langle\langle\Psi_2, \Psi_2 * \Psi_2\rangle\rangle, \quad (3.16)$$

each term on the RHS of (3.16) is given by

$$\langle\langle\Psi_2, \Psi_1 * \Psi_1\rangle\rangle = \int_0^\infty \int_0^\infty \int_0^\infty dt_1 dt_2 dt_3 e^{-t_1-t_2-t_3} \mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_3 \langle\langle B\gamma^2 \Omega^{\lambda t_1} c \Omega^{\lambda t_2} c \Omega^{\lambda t_3} \rangle\rangle, \quad (3.17)$$

$$\langle\langle\Psi_2, \Psi_2 * \Psi_1\rangle\rangle = 2\lambda \int_0^\infty \int_0^\infty \int_0^\infty dt_1 dt_2 dt_3 e^{-t_1-t_2-t_3} \mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_3 \langle\langle B\gamma^2 \Omega^{\lambda t_1} c B K c \Omega^{\lambda t_2} c \Omega^{\lambda t_3} \rangle\rangle, \quad (3.18)$$

$$\langle\langle\Psi_2, \Psi_2 * \Psi_2\rangle\rangle = 3\lambda^2 \int_0^\infty \int_0^\infty \int_0^\infty dt_1 dt_2 dt_3 e^{-t_1-t_2-t_3} \mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_3 \langle\langle B\gamma^2 \Omega^{\lambda t_1} c B K c \Omega^{\lambda t_2} c B K c \Omega^{\lambda t_3} \rangle\rangle. \quad (3.19)$$

For the correlators to be nonzero, they must have a ϕ -momentum equal to -2 , since the picture changing operator has a ϕ -momentum equal to -4 , the first term on the RHS of (3.16) gives vanishing result.

The expression for the correlators $\langle\langle B\gamma^2 \Omega^{\lambda t_1} c \Omega^{\lambda t_2} c \Omega^{\lambda t_3} \rangle\rangle$, $\langle\langle B\gamma^2 \Omega^{\lambda t_1} c B K c \Omega^{\lambda t_2} c \Omega^{\lambda t_3} \rangle\rangle$ and $\langle\langle B\gamma^2 \Omega^{\lambda t_1} c B K c \Omega^{\lambda t_2} c B K c \Omega^{\lambda t_3} \rangle\rangle$ can be derived using the techniques developed in the appendix. Plugging the expression for these correlators into equations (3.17)–(3.19), applying the change of variables as in [25] $t_1 \rightarrow uv_1$, $t_2 \rightarrow uv_2$, $t_3 \rightarrow u(1 - v_1 - v_2)$ and performing the v_1, v_2 integral, we obtain from (3.16)

$$\begin{aligned} \langle\langle\Psi_\lambda, \Psi_\lambda * \Psi_\lambda\rangle\rangle &= \frac{1}{4\pi^2} \int_0^\infty du e^{-u} [6(\lambda - 1)(2\lambda - 1)u^2 - 2\lambda(4\lambda - 3)u^3 + \lambda^2 u^4] \\ &= \frac{3}{\pi^2}. \end{aligned} \quad (3.20)$$

As it was expected, we see that the value of the cubic term does not depend on the parameter λ . This result (3.20) proves the statement that the equation of motion, in the modified cubic superstring field theory, is satisfied when it is contracted with the regularized solution itself.

3.3 \mathcal{L}_0 level expansion

Although we have the expected result for the value of the kinetic (3.14) and cubic term (3.20), we would like to confirm our calculation by using the \mathcal{L}_0 level expansion of the solution. In order to evaluate the kinetic and cubic term of the modified cubic superstring field theory action in the \mathcal{L}_0 level expansion scheme, let us write the regularized solution (3.5) in terms of \mathcal{L}_0 eigenstates

$$\begin{aligned} \Psi_\lambda = & \sum_{n,p} f_{n,p}(\lambda) (\mathcal{L}_0 + \mathcal{L}_0^\dagger)^n \tilde{c}_p |0\rangle + \sum_{n,p,q} f_{n,p,q}(\lambda) (\mathcal{B}_0 + \mathcal{B}_0^\dagger) (\mathcal{L}_0 + \mathcal{L}_0^\dagger)^n \tilde{c}_p \tilde{c}_q |0\rangle \\ & + \sum_{n,t,u} g_{n,t,u}(\lambda) (\mathcal{B}_0 + \mathcal{B}_0^\dagger) (\mathcal{L}_0 + \mathcal{L}_0^\dagger)^n \tilde{\gamma}_t \tilde{\gamma}_u |0\rangle, \end{aligned} \quad (3.21)$$

$$\begin{aligned} f_{n,p}(\lambda) = & 2^{-n} \frac{(\lambda - 1)}{\lambda} \left(\frac{1}{n!} + \frac{\lambda}{(n-1)!} \right) \delta_{1,p} \\ & + \frac{2^{-n+p-1}}{\lambda} \sum_{k=0}^n \frac{(-1)^{n-k} \lambda^{n-k-p+1} (n-k-p+1)!}{k! (n-k)!} \\ & + 2^{-n+p-1} \sum_{k=0}^{n-1} \frac{(-1)^{n-k-1} \lambda^{n-k-p} (n-k-p)!}{k! (n-k-1)!}, \end{aligned} \quad (3.22)$$

$$\begin{aligned} f_{n,p,q}(\lambda) = & \frac{2^{-n-2}(1-\lambda)}{n!} (\delta_{0,q} \delta_{1,p} - \delta_{0,p} \delta_{1,q}) \\ & + 2^{-n+p+q-3} (q-p) \sum_{k=0}^n \frac{(-1)^{n-k} \lambda^{n-k-p-q+1} (n-k-p-q+1)!}{k! (n-k)!}, \end{aligned} \quad (3.23)$$

$$\begin{aligned} g_{n,t,u}(\lambda) = & 2^{-n+t+u-2} \sum_{k=0}^n \frac{(-1)^{n-k} \lambda^{n-k-t-u+1} (n-k-t-u+1)!}{k! (n-k)!} \\ & + (\lambda - 1) 2^{-n+t+u-2} \sum_{k=0}^{n-1} \frac{(-1)^{n-k-1} \lambda^{n-k-t-u} (n-k-t-u)!}{k! (n-k-1)!} \\ & + (1 - \lambda) (1 - t - u) 2^{-n+t+u-2} \sum_{k=0}^n \frac{(-1)^{n-k} \lambda^{n-k-t-u} (n-k-t-u)!}{k! (n-k)!}. \end{aligned} \quad (3.24)$$

Next we follow the same steps developed in the bosonic case. Replacing the \mathcal{L}_0 level expansion of the regularized solution Ψ_λ (3.21) with $z^{\mathcal{L}_0} \Psi_\lambda$ and plugging it into the kinetic

term, we arrive to

$$\langle\langle\Psi_\lambda, z^{\mathcal{L}_0^\dagger} Q z^{\mathcal{L}_0} \Psi_\lambda\rangle\rangle = -\frac{2}{\pi^2 z^2} + \left(\frac{2\lambda}{\pi^2} - \frac{2}{\pi^2}\right) \frac{1}{z} + \left(\frac{1}{\pi^2} - \frac{2\lambda}{\pi^2}\right). \quad (3.25)$$

Since equation (3.25) has a finite number of terms, there is no need for Padé approximants, therefore setting $z = 1$ from equation (3.25) we obtain

$$\langle\langle\Psi_\lambda, Q\Psi_\lambda\rangle\rangle = -\frac{2}{\pi^2} + \left(\frac{2\lambda}{\pi^2} - \frac{2}{\pi^2}\right) + \left(\frac{1}{\pi^2} - \frac{2\lambda}{\pi^2}\right) = -\frac{3}{\pi^2}. \quad (3.26)$$

As we have noticed, in the superstring case the power series expansion of the kinetic term (3.25) has a finite number of terms, this result is in contrast to the bosonic case, where the series has an infinite number of terms (2.29). The reason for this result is due to the fact that the correlation functions in the superstring case are much simpler than the bosonic ones⁶. As it is summarized in appendix A, in the bosonic case the expressions for the correlators are given in terms of trigonometrical functions, while in the superstring case the expressions for the correlators are given in terms of polynomials.

To evaluate the cubic term using its formal power series expansion in z , we follow the same steps developed in the case of the kinetic term. Plugging the \mathcal{L}_0 level expansion of the regularized solution (3.21) into the cubic term, we get

$$\langle\langle\Psi_\lambda, z^{\mathcal{L}_0^\dagger} (z^{\mathcal{L}_0} \Psi_\lambda) * (z^{\mathcal{L}_0} \Psi_\lambda)\rangle\rangle = \frac{9}{2\pi^2 z^2} - \frac{3\lambda}{\pi^2 z} + \left(\frac{3\lambda}{\pi^2} - \frac{3}{2\pi^2}\right). \quad (3.27)$$

Since this last equation (3.27) has a finite number of terms, as in the case of the kinetic term, there is no need for Padé approximants, therefore setting $z = 1$ from equation (3.27) we obtain

$$\langle\langle\Psi_\lambda, \Psi_\lambda * \Psi_\lambda\rangle\rangle = \frac{9}{2\pi^2} - \frac{3\lambda}{\pi^2} + \left(\frac{3\lambda}{\pi^2} - \frac{3}{2\pi^2}\right) = \frac{3}{\pi^2}. \quad (3.28)$$

4 Summary and discussion

We have shown that our recently proposed identity based solutions [1], in open bosonic string field theory as well as in the modified cubic superstring field theory, can be consistently regularized. By consistent we mean that the resulting regularized solution brings the right value for the kinetic and cubic term of the string field theory action.

We have proved the correctness of the above statement by employing two different means: straightforward analytical computations and by using the \mathcal{L}_0 level expansion of the solution. It turns out that, in the bosonic case, employing the \mathcal{L}_0 level expansion scheme

⁶Let us point out that a similar result was found in [26] for the original Erler's solution.

the use of Padé approximants was needed, while in the superstring case the expected value for the kinetic and cubic term was derived without the use of Padé approximants. As a consequence of these results, we proved that the assumption of the validity of the equation of motion when contracted with the regularized solution itself was nevertheless correct.

It would be important to extend this analysis to the case of Berkovits WZW-type superstring field theory [22], since this theory has a non-polynomial action, the issue for finding the tachyon vacuum solution and the computation of the value of the D-brane tension seems to be highly cumbersome. Nevertheless, we hope that the ideas developed in this paper should be very useful in order to solve this challenging puzzle.

One more significant application of the techniques established in this paper, as discussed in [25], should be the extension of the subalgebra generated by the basic string fields K , B , c and γ in order to analyze identity based solutions in more general string field configurations [27, 28], such that multiple D-branes, marginal deformations, lump solutions as well as time dependent solutions.

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A Correlation functions

In this appendix we provide the details related to the derivation of the correlators used in equations (2.14), (2.21)–(2.23), (3.11)–(3.13) and (3.17)–(3.19). Let us start with the correlators

$$\langle c\Omega^{\lambda t_1} cK c\Omega^{\lambda t_2} \rangle = -\partial_{s_1} \left[\langle c(s_1 + t_1\lambda + t_2\lambda) c(s_1 + t_2\lambda) c(t_2\lambda) \rangle_{s_1+t_1\lambda+t_2\lambda} \right] \Big|_{s_1=0}, \quad (\text{A.1})$$

$$\langle c\Omega^{\lambda t_1} c\Omega^{\lambda t_2} c\Omega^{\lambda t_3} \rangle = \langle c(t_1\lambda + t_2\lambda + t_3\lambda) c(t_2\lambda + t_3\lambda) c(t_3\lambda) \rangle_{t_1\lambda+t_2\lambda+t_3\lambda}, \quad (\text{A.2})$$

where the expression for the correlator $\langle c(x_1)c(x_2)c(x_3) \rangle_L$ is given by

$$\langle c(x_1)c(x_2)c(x_3) \rangle_L = \frac{L^3}{\pi^3} \sin\left(\frac{\pi(x_1 - x_2)}{L}\right) \sin\left(\frac{\pi(x_1 - x_3)}{L}\right) \sin\left(\frac{\pi(x_2 - x_3)}{L}\right). \quad (\text{A.3})$$

For the computation of the firsts two correlators (A.1) and (A.2), the correlator (A.3) is all we need, for instance using (A.3) from (A.1) we obtain

$$\langle c\Omega^{\lambda t_1} cK c\Omega^{\lambda t_2} \rangle = -\frac{\lambda^2 (t_1 + t_2)^2}{\pi^2} \sin^2\left(\frac{\pi t_1}{t_1 + t_2}\right). \quad (\text{A.4})$$

The next two correlators, which were used in the computation of the cubic term of the open bosonic string field theory action, are given by

$$\langle cBKc\Omega^{\lambda t_1}cBKc\Omega^{\lambda t_2}c\Omega^{\lambda t_3} \rangle = \partial_{s_1}\partial_{s_2} \left[\langle c(\alpha_1)Bc(\alpha_2)c(\alpha_3)Bc(\alpha_4)c(\alpha_5) \rangle_{(t_1+t_2+t_3)\lambda+s_1+s_2} \right] \Big|_{s_1=0, s_2=0}, \quad (\text{A.5})$$

$$\langle cBKc\Omega^{\lambda t_1}c\Omega^{\lambda t_2}c\Omega^{\lambda t_3} \rangle = -\partial_{s_1} \left[\langle c(\beta_1)Bc(\beta_2)c(\beta_3)c(\beta_4) \rangle_{(t_1+t_2+t_3)\lambda+s_1} \right] \Big|_{s_1=0}, \quad (\text{A.6})$$

where

$$\begin{aligned} \alpha_1 &= (t_1 + t_2 + t_3)\lambda + s_1 + s_2, & \alpha_2 &= (t_1 + t_2 + t_3)\lambda + s_2, \\ \alpha_3 &= (t_2 + t_3)\lambda + s_2, & \alpha_4 &= (t_2 + t_3)\lambda, & \alpha_5 &= t_3\lambda, \\ \beta_1 &= (t_1 + t_2 + t_3)\lambda + s_1, & \beta_2 &= (t_1 + t_2 + t_3)\lambda, \\ \beta_3 &= (t_2 + t_3)\lambda, & \beta_4 &= t_3\lambda. \end{aligned} \quad (\text{A.7})$$

The correlators $\langle c(\alpha_1)Bc(\alpha_2)c(\alpha_3)Bc(\alpha_4)c(\alpha_5) \rangle_L$ and $\langle c(\beta_1)Bc(\beta_2)c(\beta_3)c(\beta_4) \rangle_L$ can be computed using the following correlator [12]

$$\begin{aligned} \langle Bc(x_1)c(x_2)c(x_3)c(x_4) \rangle_L &= \frac{x_1}{L} \langle c(x_2)c(x_3)c(x_4) \rangle_L - \frac{x_2}{L} \langle c(x_1)c(x_3)c(x_4) \rangle_L \\ &+ \frac{x_3}{L} \langle c(x_1)c(x_2)c(x_4) \rangle_L - \frac{x_4}{L} \langle c(x_1)c(x_2)c(x_3) \rangle_L. \end{aligned} \quad (\text{A.8})$$

In the case of the modified cubic superstring field theory, the expressions for the correlators are much easier than the ones given in the bosonic case [26]

$$\langle \langle c\Omega^{\lambda t_1}\gamma^2\Omega^{\lambda t_2} \rangle \rangle = \frac{\lambda^2 (t_1 + t_2)^2}{2\pi^2}, \quad (\text{A.9})$$

$$\langle \langle c\Omega^{\lambda t_1}cBK\gamma^2\Omega^{\lambda t_2} \rangle \rangle = -\frac{\lambda t_1}{2\pi^2}, \quad (\text{A.10})$$

$$\langle \langle cBKc\Omega^{\lambda t_1}cBK\gamma^2\Omega^{\lambda t_2} \rangle \rangle = 0, \quad (\text{A.11})$$

$$\langle \langle B\gamma^2\Omega^{\lambda t_1}c\Omega^{\lambda t_2}c\Omega^{\lambda t_3} \rangle \rangle = \frac{\lambda^2 t_2 (t_1 + t_2 + t_3)}{2\pi^2}, \quad (\text{A.12})$$

$$\langle \langle B\gamma^2\Omega^{\lambda t_1}cBKc\Omega^{\lambda t_2}c\Omega^{\lambda t_3} \rangle \rangle = -\frac{\lambda t_2}{2\pi^2}, \quad (\text{A.13})$$

$$\langle \langle B\gamma^2\Omega^{\lambda t_1}cBKc\Omega^{\lambda t_2}cBKc\Omega^{\lambda t_3} \rangle \rangle = 0. \quad (\text{A.14})$$

To derive these correlators, we have used the following two basic correlators ⁷

$$\langle \langle c(x_1)\gamma^2(x_2) \rangle \rangle_L = \frac{L^2}{2\pi^2}, \quad (\text{A.15})$$

$$\langle \langle Bc(x_1)c(x_2)\gamma^2(x_3) \rangle \rangle_L = \frac{L(x_1 - x_2)}{2\pi^2}. \quad (\text{A.16})$$

⁷These correlation functions has been computed using the normalization: $\langle \xi(x)c\partial c\partial^2 c(y)e^{-2\phi(z)} \rangle = 2$.

B Padé approximant computations

In this appendix, as a pedagogical illustration of the numerical method based on Padé approximants, we are going to compute in detail the normalized value of the kinetic term $\frac{\pi^2}{3} \langle \Psi_\lambda, z^{\mathcal{L}_0^\dagger} Q z^{\mathcal{L}_0} \Psi_\lambda \rangle$ at order $n = 2$. At this order we need to consider terms in the series expansion of the kinetic term (2.29) up to quadratic order in z , namely

$$-\frac{4}{\pi^2 z^2} + \frac{(\pi^2 - 4)(2\lambda - 1)}{2\pi^2} + \frac{\pi^2(\lambda^2 - 2\lambda^3)}{8} z^2. \quad (\text{B.1})$$

Using the numerical method based on Padé approximants, first we express (B.1) as the following rational function

$$P_{2+2}^2(z) = \frac{1}{z^2} \left[\frac{a_0 + a_1 z + a_2 z^2}{1 + b_1 z + b_2 z^2} \right]. \quad (\text{B.2})$$

Expanding the right hand side of (B.2) around $z = 0$, we get up to quadratic order in z

$$\begin{aligned} P_{2+2}^2(z) &= \frac{a_0}{z^2} + \frac{a_1 - a_0 b_1}{z} + (a_2 - a_1 b_1 + a_0 b_1^2 - a_0 b_2) \\ &+ (a_1 b_1^2 - a_2 b_1 - a_0 b_1^3 - a_1 b_2 + 2a_0 b_1 b_2) z \\ &+ (a_2 b_1^2 - a_1 b_1^3 + a_0 b_1^4 - a_2 b_2 + 2a_1 b_1 b_2 - 3a_0 b_1^2 b_2 + a_0 b_2^2) z^2. \end{aligned} \quad (\text{B.3})$$

Equating the coefficients of z^{-2} , z^{-1} , z^0 , z^1 , z^2 in equations (B.1) and (B.3), we get a system of five algebraic equations for the unknown coefficients a_0 , a_1 , a_2 , b_1 and b_2 . Solving these equations we get

$$a_0 = -\frac{4}{\pi^2}, \quad (\text{B.4})$$

$$a_1 = 0, \quad (\text{B.5})$$

$$a_2 = \frac{\pi^2(8 - 16\lambda) + 32\lambda + \pi^4(-2\lambda^2 + 2\lambda - 1) - 16}{2\pi^2(-4 + \pi^2)}, \quad (\text{B.6})$$

$$b_1 = 0, \quad (\text{B.7})$$

$$b_2 = \frac{\pi^4 \lambda^2}{4(-4 + \pi^2)}. \quad (\text{B.8})$$

Replacing the value of the coefficients (B.4), (B.5), (B.6), (B.7) and (B.8) into the definition of $P_{2+2}^2(z)$ (B.2), and evaluating this at $z = 1$, we get the following normalized value for the kinetic term,

$$\frac{\pi^2}{3} P_{2+2}^2(z = 1) = \frac{-32\pi^2\lambda + 64\lambda + \pi^4(-4\lambda^2 + 4\lambda - 2) + 32}{3(\pi^4\lambda^2 + 4\pi^2 - 16)}. \quad (\text{B.9})$$

Now we are ready to evaluate this expression (B.9) for some particular values of λ . Our first interest is the case $\lambda \rightarrow 0$ which corresponds to the identity based solution, where we get

$$\frac{\pi^2}{3}P_{2+2}^2(z=1; \lambda \rightarrow 0) = -2.311600733514, \quad (\text{B.10})$$

while in the case of $\lambda \rightarrow 1$ which corresponds to the Erler-Schnabl's solution, we obtain

$$\frac{\pi^2}{3}P_{2+2}^2(z=1; \lambda \rightarrow 1) = -1.143337106188. \quad (\text{B.11})$$

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